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Note on "If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?", by Peter R. Saulson [Am. J. Phys. 65 (6), 501-505 (1997)

W. L. Andersen*

Eastern New Mexico University

Portales, NM

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A very natural and subtle question concerning the detection of gravitational waves which can be easily appreciated even by non–physicists is: "If the detector arms and wavelengths are stretched, will not an interferometer fail to show evidence of the passing wave?". This question has been answered in the literature with varying degrees of sophistication, each approach having merits of its own. For example, Garfinkle's analysis[3] is not only elegant but also applies to realistic waves while the approach of Saulson[4], although dealing with a rather unrealistic Heaviside step function wave front, requires only a visualization of the wave motion. Other worthy responses exist in the literature.[1,2] It is the purpose of this note to point out that Saulson's model can also be understood using a spacetime diagram. This has the advantage of eliminating the need for animating the wave motion mentally.

We measure time in nanoseconds and distance in units of feet defined so that the speed of light is preciously one. Light may then be adequately represented by equations of the form

$$\Psi = \cos(\omega t \pm kx) \tag{1}$$

Figure 1 shows a static depiction of such a wave with $\omega = k = 1$. In viewing this picture, one must avoid the temptation to animate the figure and view it as a snapshot in time. To achieve a snapshot one picks a line of simultaneity and imagines the cross-section of the wave cut by this line. In fact, by imagining the lines of simultaneity of two different observers one can readily derive the relativistic doppler effect in a geometric fashion.

Using the customary description of gravitational waves, distances are stretched by a factor $\sqrt{1+h}$. We employ Saulson's pedagogical device of a Heaviside gravitational wave of the form $h(t) = h_0 H(t - \tau)$. In figure 2 the gravitational wave hits at $\tau = 4\,\text{ns}$, just before the radiation emitted at $t = 0\,\text{ns}$ reaches the mirror. For easily visualization, we take the wave amplitude to be quite large with $h_0 = 9/16$ so that $\sqrt{1+h_0} = 5/4$.

The details of inteferometer design are simplified for convenience rather that realism. We suppose that a source positioned $x = 0$ emits electromagnetic waves (which we call radiation) of frequency 1 GHz in the $+x$ direction. The beam splitter is at $x = 0$ allowing half of the radiation through to the mirror stationed at the coordinate $x = 4\,\text{ft}$. Figure 2 also shows diagonal lines of constant phase and may be taken to represent wave crests. We orient the splitter normal to the beam so that we are comparing the phase of the reflected

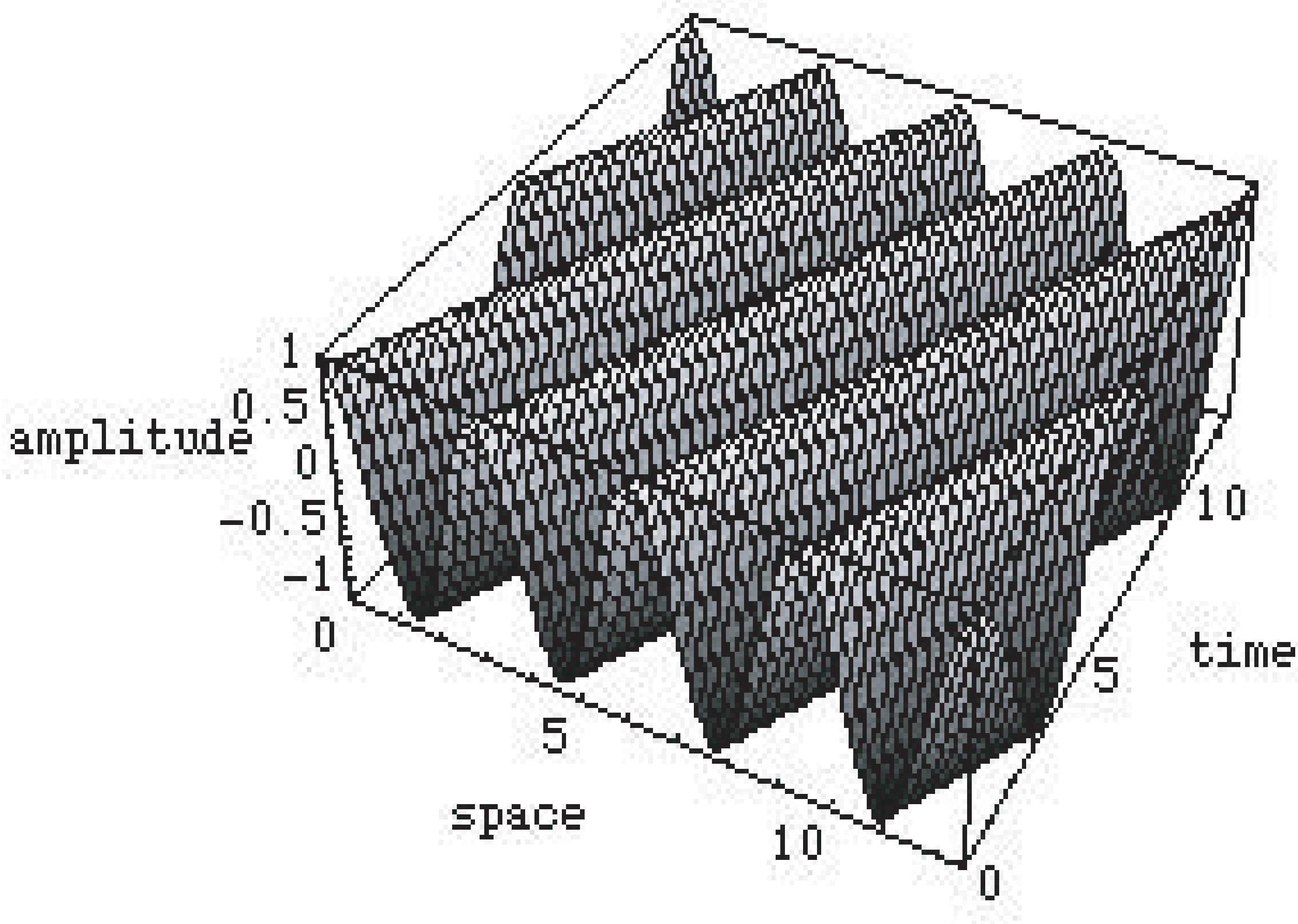

FIG. 1: A wave in spacetime.

wave directly with the phase at the source. Arrows show how the position of the radiation wave crests change at $t = 4\,\mathrm{ns}$ when the gravitational wave arrives. For clarity, radiation emitted before $t = 0\,\mathrm{ns}$ is not shown. Note that the radiation emitted at $t = 0\,\mathrm{ns}$ would have returned to $x = 0\,\mathrm{ft}$ at $t = 8\,\mathrm{ns}$ (instead of $t = 9\,\mathrm{ns}$) had there been no gravitational wave.

Once the details of the diagram are understood, understanding the resolution of the paradox is a merely a matter of inspection. Prior to arrival of the gravitational wave the

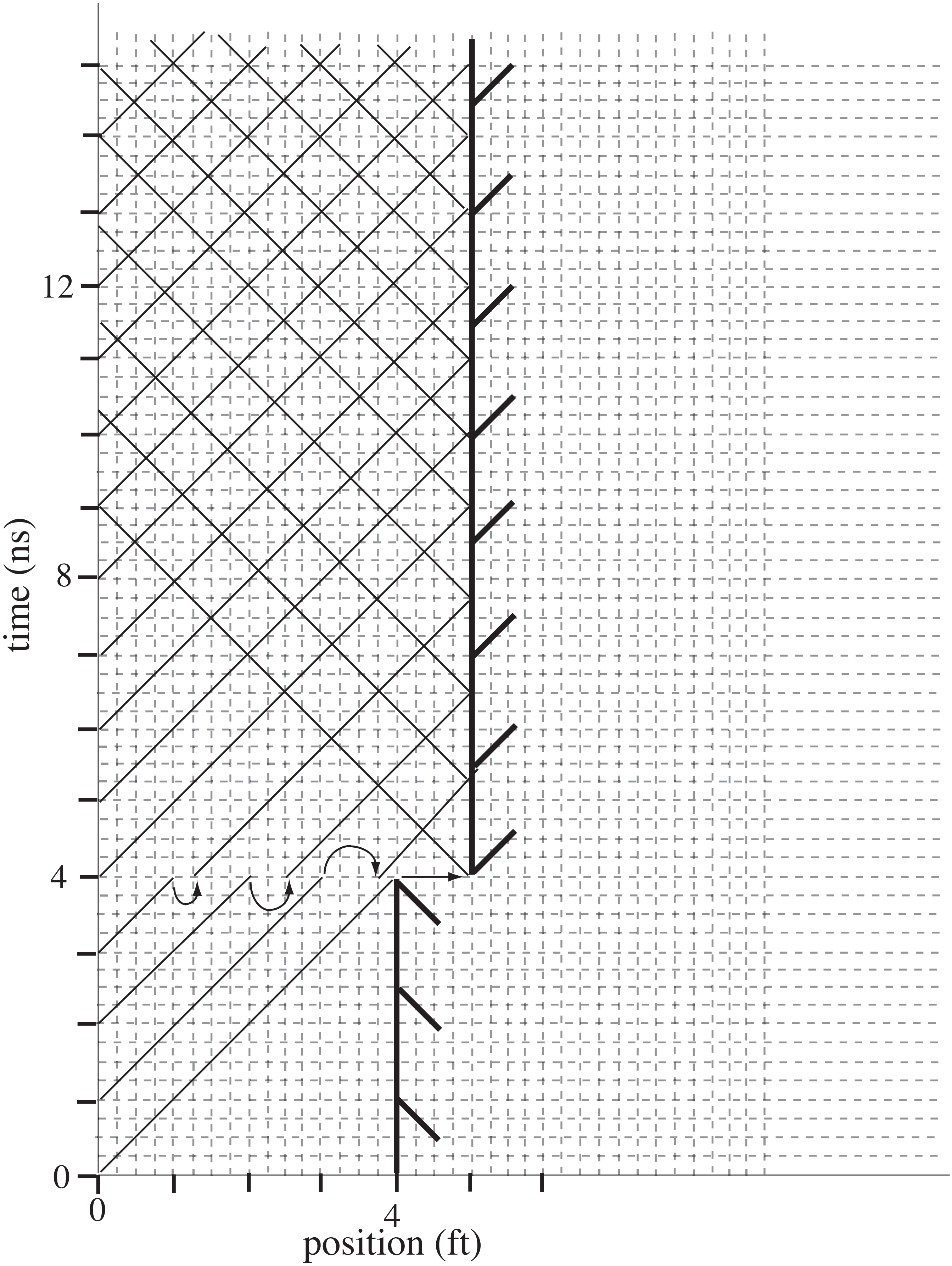

FIG. 2: Lines of constant phase in spacetime. The beam splitter is at $x = 0$ and a mirror at $x = 4\,\mathrm{ft}$. A Heaviside gravitational wave hits at $4\,\mathrm{ns}$. It is important to note that the horizontal axis represents physical distance from $x = 0$ rather than coordinate distance.

reflected radiation returns to $x = 0$ in phase with the radiation passing through $x = 0$ in the $+x$ direction. Notice that wave crests that are between the coordinates $x = 0$ and $x = 4\,\mathrm{ft}$ when the gravitatonal wave arrive are returned to $x = 0$ with successively increasing phase delays compared to the source. This spacetime diagram communicates Saulson's argument in a visually static fashion which may clarify the analysis for some.

Acknowledgments

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* `William.Andersen@enmu.edu`

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